PI-RATE

THE WELHAM MATHEMATICS MAGAZINE

THE HOUSE ALWAYS WINS!

WHY DID INDIA LOSE AGAINST NEW ZEALAND IN THE ICC SEMI FINALS?

FOUNDERS' ISSUE, 2019

INSIDE: WHICH QUEUE TO STAND BEHIND?

AN INSIGHT INTO JACKPOT HUNTING

FROM THE EDITOR'S DESK

Dear Readers,

Logic and mathematics are nothing but specialized linguistic structures. Math is not only limited to the strings of numbers. It may not teach us how to perform a task but it surely gives us every reason to hope that every problem has a well proven solution. Its universal validity and capacity to provide a logical explanation for the behavior and existence of each particle in the cosmos is astounding. What is even more wonderful about math is that it doesn't exist in a physical dimension. It exists entirely in the human mind; it's a mental concept, which holds true everywhere. However, the pre-conceived notion about math is that it is a boring and technical subject.

As the Editor-in-chief this year, my main objective was to make Pi-rate more reader friendly and to reach out to the masses. This herculean task would not have been possible without the wonderful editorial team which shared the same sense of passion and vision that I had and worked tirelessly to achieve it.

In this issue we have done away with the deep contextual math. We have tried to explore mathematics outside the textbooks and have presented it in an elementary and engaging manner. Welhamites have taken a crack at the mathematical application behind Instagram and to something as simple as to decide the best queue to line up behind. We have also presented the odds behind the controversial impeachment of the US president. We also have an interesting cover story in which we have discussed the role of math in casinos.

Through this humble initiative it's my endeavour to let the budding young minds of Welham roam free in the realm of logic and reason and to look at the beauty of mathematics. It gives me immense pleasure to present the all new version of the Pi-rate on the 62nd Founders'.

Compiling a magazine is a gigantic task and it would not have been possible without the editorial board and the unending support of Mr.Rawat who helped this issue take form.

Happy reading!

Ishika Agarwal
Editor-in-chief

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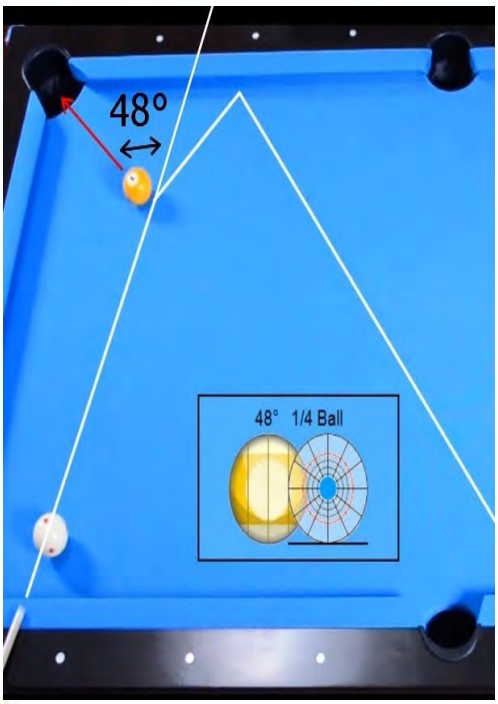
PLAY POOL LIKE A MATHEMATICIAN

BY VAANYA SINGH

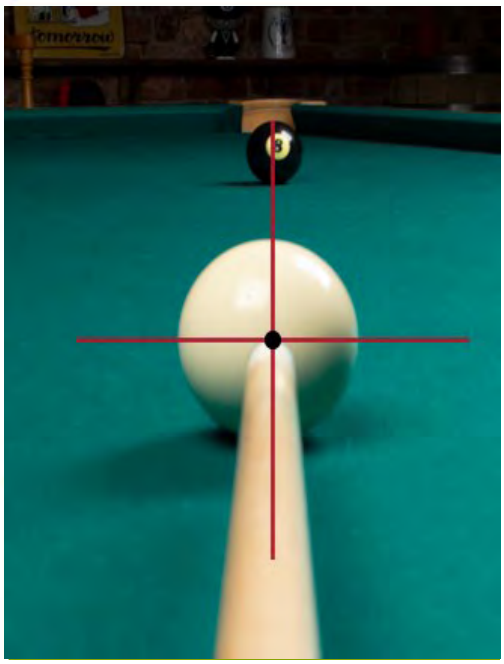
The mathematical study of pool generally consists of reflections in which the reflection and incidence angles are the same. The game mainly consists of two types of shots-the angle shot and straight shot.

Most pool shots are an angle shot. The degree of difficulty depends on the angles involved as the larger the angle the harder it is to make the shot. Intermediate and advanced pool players use "The Law of Reflection." The law states that a ball will bounce off the side of the table at the same angle at which it hits the rail. For example, if a ball hits the side of the table or the rail at an angle of 40 degrees, it will bounce off the rail at an opposite angle of 40 degrees. With the help of this law a player can predict where the ball will go based on the angle it makes.



"Straight shots are usually the easiest shot to make, but here the degree of difficulty depends on the distance between the three objects we consider."

The other type of shot is a "straight shot". Three objects are considered in this type of shot, the cue ball, the object ball and the pocket. The player hits the cue ball into the object ball which directly goes into the pocket, without bouncing off the rail. In other words, the cue ball, object ball, and pocket are in a straight line. Straight shots are usually the easiest shot to make, but here the degree of difficulty depends on the distance between the three objects we consider. The greater the distance is between objects, errors like the side-spin gets amplified.



Famous mathematician Rick Mabry used trigonometry to find the most difficult straight shot, which is when the distance from the pocket to the cue ball is 1.618 times the distance from the pocket to the object ball. You may recognize this famous number as Phi, or the golden ratio. Thus if one wants to play pool like a mathematician then this is the right path.

KAREN UHLENBECK

Abel prize awardee

BY ANURATI SHARMA

"*I find that I am bored with anything I understand*", this line said by Karen is the main reason for her success.

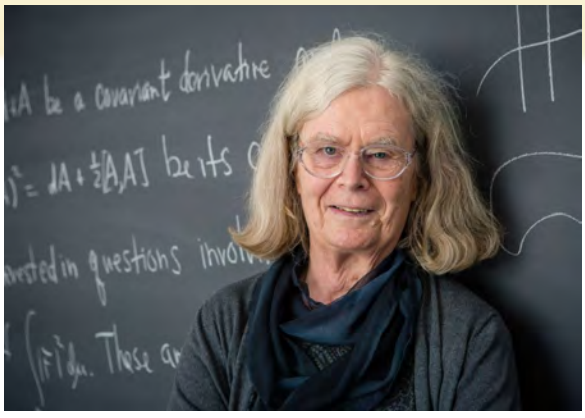
Karen Uhlenbeck is the first woman to receive an Abel prize in Mathematics. Her curiosity, suggested through her quote, shows us her eagerness in work. She is a woman who never gets tired of the amount of knowledge she gains and the effort which is put into gaining it. Her pioneering achievements in geometric partial differential equations, gauge theory and her integral systems made her worthy of the Abel prize.

Apart from all of this she has many achievements to her credit, such as co-founding the Institute of Advanced Studies in Princeton, England, the program for women and Mathematics in order to encourage women to contribute in more research of mathematics.

Also being a feminist, she has often criticized the culture of many mathematicians that sees minimal involvement of women. She believes that if women take more part in mathematics it will lead to empowerment sooner. Being the first woman to receive the highest honor in the field of mathematics is indeed a great achievement, the legacy of which must be carried forward by young women all over the world.

Currently this wondrous professor, aged 77, is teaching at the National University of Texas, Austin. She hopes that more women excel at the things that they are good at so that one day they can contribute to the world in their own possible way.

"She is a woman who never gets tired of the amount of knowledge she gains and the effort which is put into gaining it."



"REEL" ACTION, REAL BETS

BY NAINA RASTOGI

We have all tried our luck on slot machines, crossing our fingers and hoping that indeed, luck is on our side. But maybe there is more to slot machines than luck?

To explain the working of a slot machine, let us take an example of a machine with three reels with 6 different symbols on each reel. But as shown the table alongside, the number of each symbol on a reel might vary.

Symbol	Frequency (Reel 1)	Frequency (Reel 1)	Frequency (Reel 3)
BAR BAR BAR	1	1	1
7	3	1	1
	1	3	3
	4	4	4
	4	4	4
	4	4	4
Total frequency on each reel	17	17	17

It is quite obvious that the total number of outcomes of getting a symbol in a series is given by the number of times it appears in

: Reel 1 X No. in Reel 2 X No. in reel 3. So the number of ways to get three 7 symbols would be 3x1x1 = 3. Likely the number of times to get the sequence melon-melon-bell would be 1x3x4=12. Calculating this for each winning combination, we can find that **Σ** (winning probability) is 205.

Having seventeen results on each reel gives us 17x17x17= 4913 different combinations.

Therefore, the probability of getting the same symbol on all three, that is, the winning probability would be, 205/4913, which is approximately 4.21%.

There are many different types of slot machines, some machines have 5 reels instead of three, which increases the number of combinations for a 20 symbol reel machine to 20^5 which is equal to 32,00,000.

To compensate for this there are added ways of winning, but those can be saved for another day.

One thing the players can be sure of is the fact that if slot machines never paid out anything, players would stop playing, and casinos would run out of business. Most jurisdictions mandate that slots return a set minimum amount to players (85 percent is the magic number in Nevada, though most machines return more than that on average).

It might not be much comfort when you've lost your last credit, but someone has to win a piece of that 85 percent, and next time it might be you.

But something to keep in mind is that no matter how lucky you feel, eventually the casino will get luckier.

TRUMP-ED?

BY JANHVI CHANDRA

We have been constantly hearing a lot about the ongoing impeachment process in the United States and as many political pundits might predict, there is a way to affirm if the Donald Trump will be removed from the White House. The process is rather simple to understand. Both the houses must pass the resolution by two-thirds majority with 218 votes out of 538 in the House of Representatives and 67 votes out of 100 in the Senate.

There is a field of mathematics called 'Combinatorics' that deals with problems of selection, arrangement and operation within a finite or discrete system. It's a matter of probability that allows us to predict what the outcome is likely to be.

At present the possibility of impeachment is very slim as Republicans dominate both chambers but a lot could change post the mid-term elections taking place in November for the lower house. To make this clearer, we can assume that all democrats will surely vote to impeach in the house and declare him guilty in the senate. This would mean that we can zero down on one factor to determine his impeachment: the chances of any republican voting against Trump. Moreover, this decision to vote against trump would be independent and not influenced by any other congressman. We can call this probability 'P'.

Let us also assume that republicans maintain a very narrow margin in the house with just 218-217 majority. The congressman crossing over from republican could be X, Y or Z. Ultimately, there are 218 ways in which a candidate from republicans will vote against the president while all other 217 maintain their party stance. When we calculate, we infer that if X decided to vote trump guilty then others will stay loyal with the probability 1-P. The probability for the others would be P (1-P)… till 217(1-P) in the product. If both, X and Y, decide to cross over then the probability will be 216(1-P) for those who stay loyal and the same process is repeated as more and more congressmen cross over. This is when we use combinatorics until we reach an expression that gives us the odds of the impeachment where the last variable is only P.

From this we can understand that even if a small number of republicans cross over, there is a high possibility that the impeachment will be carried forward to the senate.

This is where it becomes tricky as they need almost 11 senators to cross over given that there is limited democratic control in this chamber.

This makes the verdict next to impossible even if the process is successful in the lower house. Although a mathematical model is presented, the odds of this are dependent on multiple assumptions and ultimately the will of the congressman. Only time will tell if Trump will join Bill Clinton and Andrew Johnson as the next impeached president of the United States..

ROLLER COASTER

BY VAANYA AGARWAL

Why is a rollercoaster so exciting and terrifying yet safe? Well the credit goes to its designers who use the fundamental laws of mathematics while designing the rollercoaster structures and to calculate forces which give momentum to it. Designers of these models use the applications of calculus to analyze the loops, twists and curves and to measure the maximum and minimum points on the tracks. The required velocity and acceleration is also calculated with the use of derivatives and is maintained within the safety limit. To ensure that the track is smooth, the straight stretches of the rollercoaster must be tangent to the parabolas at the curving points.

Centripetal acceleration is the motion of an object moving in a circular path. The curved tracks create this force and the coaster is directed towards the center of the loop whereas the riders feel the fictitious force called the centrifugal force.

Modern designers have adopted an upside down teardrop shape of curves called clothoid in which tracks are sharply curved at the top than the bottom. This is so that most of the turn happens at the top where acceleration is the least and the coaster in itself is moving the slowest. This way less gravitational force is applied on the riders.

Roller coasters don't have engines but still need energy. Before the ride starts an electric winch winds up the coaster to the top of the hill where the energy is stored in the form of potential energy and it changes into kinetic energy at its descent. Thus, another equation applied here is that total energy is equal to potential energy plus kinetic energy.

It is also interesting to know that the thrill experienced during the drop has a mathematical aspect to it .The numeric thrill of a drop is the product of the angle of the steepest descent in the drop and the total vertical distance in the drop. Therefore, the total thrill of the rollercoaster is the sum of thrills of all drops. When you step off a coaster one feels an out-of-control experience, but in reality every twist and turn has been mathematically tested to produce maximum thrills with minimum risk.

NOT JUST A MATTER OF CHANCE!

BY ISHIKA AGARWAL

It is a well-known fact that casino gaming is one of the most regulated industries across the globe. This is quite evident; after all every night billions of dollars are put on bet here. Casino is not merely an entertainment alternative. In fact it is an extensive business line with a distinct and carefully-planned business model that will ensure profits. The designers of these models are mathematical and statistical geniuses who design the games in such a way which ensures stacking the odds in favor of the casino, thereby assuring positive gross gaming revenues.

The central question which arises is what makes casinos the hub of profits? If a straight forward approach to this is followed then the answer lies in the law of regression. Casinos work on its core advantage of its huge number of patrons. Hence, greater the number of gamblers, nearer casinos approach to profits. What if I said that all the table games, gaming machines, and random number ticket games such as keno are actually rigged? It is surprising but also true.

These games and bets are actually designed in such a way that it offers an inbuilt statistical advantage to the casino in the long run. This is called the house edge. The house edge can be computed easily for some games - roulette and craps - for others it requires more sophisticated mathematical analysis. For example in double-zero roulette, this figure is 5.3%. This means that in the long run the house will generate 5.3% of the money wagered. In the short term, of course, the actual win percentage will differ from the theoretical win percentage. In the long run it is achieved by the law of large numbers-as the number of trials gets larger, the actual win percentage gets closer to the theoretical win percentage.

But all is not bad in the house. Although all the laws of probability are in the casino's favor, the house edge varies significantly among the different casino games. The game with the lowest house edge is blackjack; if a player follows perfect betting strategy and can count the cards (though illegal); the house edge is only 0.5%. But it is of small relevance as it only applies if the player is playing the odds perfectly, which few people do.

The house advantage obviously doesn't mean that you can't win but what it does mean is that the more you play, the more math works against you and the better are the chances of you walking out of the casino with less money in your wallet than when you came in! A study by University of Vegas claims that the 23 Vegas casinos brought in over $72 million each in the 2013 fiscal year and ended up with over $5 billion of their visitors' money, altogether. That's an average of over $630,000 a day, per casino. It is quite evident that in casinos people do not get lucky or unlucky as it is already pre-planned by the statistical models. No matter how lucky you feel, eventually the casino is bound to get luckier.

LIKED

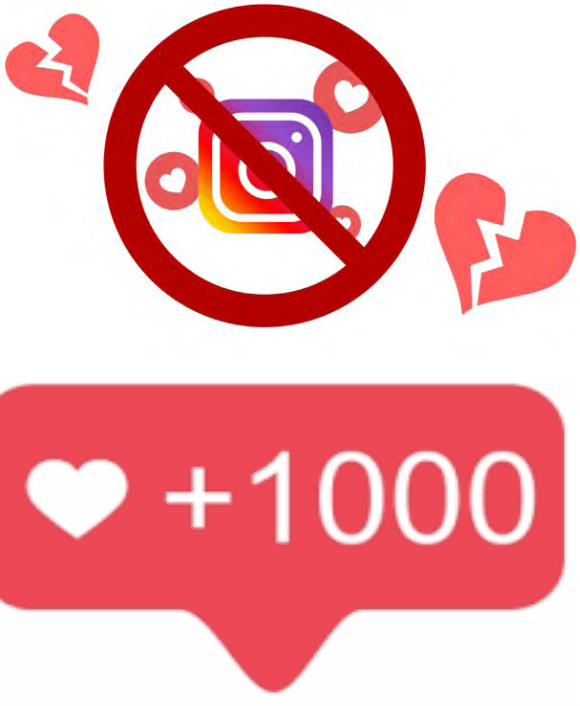
BY VANYA CHOWDHRY

Although most of us know that Instagram is an app, we don't regard it as a computer program. We tend to regard Instagram as a civic association or a sharing platform where we may or may not discover new people. What we fail to realize is that Instagram is a series of zeroes and ones, or an algorithm.

An algorithm is a mechanism which allows Instagram to observe each of its handlers, study their likes and dislikes based on certain indications, and then show them more of what they like first. This means that the manner in which Instagram works is probable. It follows definite patterns based on rules that are encrypted into its algorithm. To determine these rules, we need to discover what Instagram wants us to do.

Instagram wants us to post as many times as possible. The more active we are, the more relevant it remains. This is the core of Instagram's commercial prototype. The rules followed by this algorithm continue to reward a post if it receives enough likes within a period of certain time. On an average, the 'Like' button is pressed 4.2 billion times in a day. If a post receives 100 likes within 30 minutes, it will continue to feature on people's feed for another 30 minutes, whereas, if it attains 200 likes within an hour, the post is showcased for another hour. Once a post obtains 500 likes within five hours, according to the rule of the algorithm, it will usually continue to be seen for whole of 12 hours. If it reaches 1000 likes within 12 hours, the post usually loiters for a full day. Followed by this, at 1500 likes within 24 hours, the feed usually lingers on for another day. At over 2000 likes, it stays on for a third day.

The Instagram algorithm is predisposed to use measured calculations to determine the relevance and the influence that the users have over the others. Thus, algorithms don't only mystify the progression by which machines run the world, but also determine the dominance of the handlers.



MATH MUSICAL ~~CHAIRS~~

"There is geometry in the humming of the strings, there is music in the spacing of the spheres."- Pythagoras

BY RIYANSHI BANSAL

One may claim that listening to music enhances their cognitive skill of thinking. But it is not restricted to this. It has been observed by some mathematicians that not only listening but practicing music has helped them improve their math skills because, at some level, all music is math. Even musical notes are all about time intervals, beats per minutes and formulaic progressions.

Music is associated with something that is relaxing while math is a pure science associated with intelligence and analytical skills. It is been proven by neuroscientists that when we listen to music, different areas of our brain become engaged. When we play an instrument, then the entire brain lights up and becomes active in a way that no other activity is able to do. When children pick up learning an instrument at a young age, it helps with their cognitive development.

Many studies show a correlation on how music might benefit students struggling to keep up with their peers. In fact, Einstein used to sit and play music when he was stuck on a mathematical problem. By concentrating on the problem at hand (left brain) while playing the piano or violin (right brain), he was able to strengthen the communication between the two hemispheres of his brain and increase brain-power. Exactly how much students benefit may depend on the type of music they listen to and whether they choose to learn how to play instruments.

Some researchers claim that music activates the same areas of the brain that we use while solving spatial reasoning problems. There are certain types and frequencies of sound which are processed by the two hemispheres of the brain differently. Thus using specific music and sounds may help to simulate one hemisphere more than the other and possibly create more balance in the brain. As such, listening to music could improve a student's cognitive skills to a large extent.

THE MILLENNIUM PROBLEM

BY PALAK AGARWAL

What if I told you that you could earn a million dollars by solving a math problem? It does sound unbelievable but it is true! In 2000, the Clay Mathematics Institute of Cambridge, Massachusetts laid out seven of the most challenging problems mathematicians have ever faced and offered one million dollars to anyone who could solve even one of them.

In 2010 one of the millennium problems: The Poincaré Conjecture was solved by Grigori Perelman but six problems are still left. Out of these P vs NP problem is the easiest one to understand and explain.

P vs NP problem is a question pertaining to not only math but also computer science. Back in the 20th century computer scientists sorted out problems in classes on the basis of how fast the program could solve them. There were some programs for which they weren't sure if there was a fast way to do it. P is a class that includes all problems for which an algorithm can provide an answer in polynomial time that is in reasonable time. NP are those problems for which there is no known way to find an answer quickly and only if one is provided with the information showing the answer then it is possible to verify it quickly. Therefore this class of problems is called NP for "nondeterministic polynomial time." Sometimes we would find out that the NP problem is actually part of a P problem and then we would get a fast program to find out the solution but for a lot of NP problems that didn't seem to be happening. So people started to wonder whether all NP problems would eventually turn out to be P. That is the P vs NP question. If it is easy to check whether a solution to a problem is correct, is it also easy to solve that problem? If we can solve this question then NP-class problems such as developing accurate financial-forecasting models or analyzing protein-folding behavior in a cell would be very easy to solve and believe it or not it will change the world. P vs NP problem is just one of those 6 millennium problems. The person who solves them could be anyone, even you.

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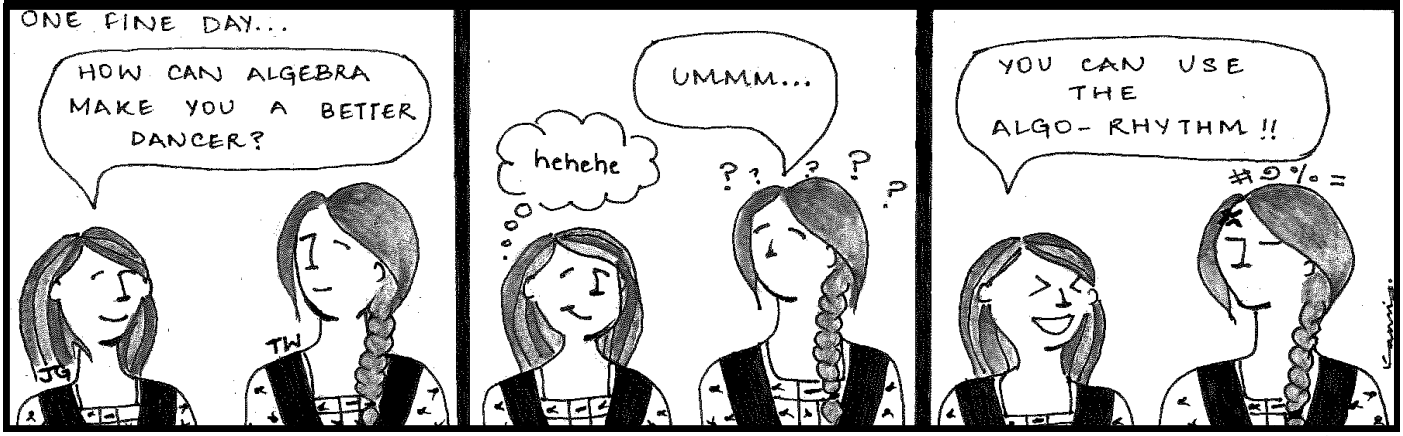


"CRICKET"-AL RAIN

BY PRISHA MANOCHA

India vs New Zealand was the much awaited Semi Final match, for which the entire nation sat with faces glued to the television and hopes soaring high for India's victory. Indians were disheartened when the match was interrupted by the rain. The Duckworth-Lewis Method was supposed to come into play in case the rain had stopped well in time. The target to be achieved according to the DLS method was a very tough one. It led to a complete change in the result of the match which was then postponed to the next day.

Duckworth-Lewis method is a mathematical method to calculate the target score for the team batting second in limited overs when the match gets interrupted by weather or other circumstances. It was worked out by Frank Duckworth and Tony Lewis after much research in order to be able to conclude the interrupted matches with a fair result. After the retirement of Duckworth and Lewis, Professor Steven Stern became the custodian, therefore it came to be known as Duckworth-Lewis-Stern method (or DLS method). It is interesting to know that a change in the weather conditions can completely change the winning probability of a team. Therefore, the DLS method was an attempt to set a statistically fair target for the second team which would be of the same difficulty as the original target.

This can be clearly explained with the example of the World Cup 2019 Semi-Final match between India and New Zealand which took place at the Old Trafford Stadium in Manchester on the 9th of July this year. The match was interrupted by rain and it was therefore assumed that the DLS method would be applied. This method was to be followed in case the rain stopped after a considerable amount of time, although in reality the match took place the next day.

Before the match was interrupted, New Zealand had scored 211 runs in 46 overs with a fall of five wickets. The target set for India according to the time reduced would have been 237 runs in 46 overs, or 223 runs in 40 overs or 209 runs in 35 overs or 192 runs in 30 overs or 172 runs in 25 overs or 148 runs in 20 overs. The target score was comparatively difficult as New Zealand had made just 211 runs in 46 overs while India was expected to make 237 runs in the same number of overs in case the DLS method had come into play.

When the match is interrupted by weather and one or both teams don't get to bat their full quota of overs, the outcome has to be reached in the time available after the resumption of the match.

ICC has tried to arrive at a formula that takes into account most parameters and reflects the efforts of both the teams equally. DLS method is thus considered to be the most accurate method for this in international cricket.

IT'S NOT YOU, IT'S THE CLAW MACHINE

BY JAHNVI GHAI

The claw machine game is pretty straight forward. Something meant for people of all age groups, the arcade game is seen almost everywhere, displaying mounds of brilliantly colored surprises, mostly stuffed toys, jewelry and candies, contained inside a Plexiglas box, only to be freed by a metal claw and the trick of a hand. More expensive prizes are usually placed in plastic bags to make it harder for the player to pick. The player inserts money or tokens into the machine, which then allows the player to control the joystick which in turn controls the claw for some amount of time, (controlled by the operator) usually for 30 seconds and very rarely a minute. At the end of the allotted time, the claw descends and tries to grab. After the attempt to grip the claw moves towards the opening in the corner of the case and drops its contents.

But maybe it takes more than just a little bit dexterity to win the prize?

Anyone who has played with a claw machine can relate to the experience of having the claw perfectly positioned, only to see it weakly graze the prize before pulling up. Some people think the claw machine is too difficult to win because the stuffed animals or prizes are packed so tightly together. But the reason is bigger and sneakier than that: the claw machine is programmed to have a strong grip only for a part of the time. Those claw machines are rigged! But they're rigged in a surprisingly clever way.

The instruction guides of the claw machines have in them a horrifying sub heading saying: *'Managing profit is made easy'.*

Simply input the coin value, the average value of the prize and the profit level or desired profit. The machine is programmed in such a way that it will automatically calculate when to send the claw at full strength. For example: if it costs 10 rupees to play the claw machine and the prize inside costs 450 rupees. To make a profit of 50% full power will be sent to the claw only about one in every 21 games.

They also randomize that winning game within a range so that players cannot predict exactly when it will happen. The machine controllers also program the machine to make you think that you almost won. Most of the times claw machines are a lot like slot machines. If the machine operators want to make the claw machine extremely unfair against the players, there's hardly anything stopping them. Most of the regulations focus on the prize size not the strength of the claw.

Now what is the probability that you will win a prize playing the claw machine?

Since the odds of winning are 1 in 21 games and each game is completely unpredictable the chances are very slim. Some people might get lucky and win on the 21st cycle. However people need to make at least 20 tries before succeeding. The chance of winning is 1 in 15 people, and this is a huge profit to the game makers and the arcade companies. So the game is receiving 2%-5% more money and profit because of this game.

The only thing you can do is try your luck now!

QUEUE THEORY

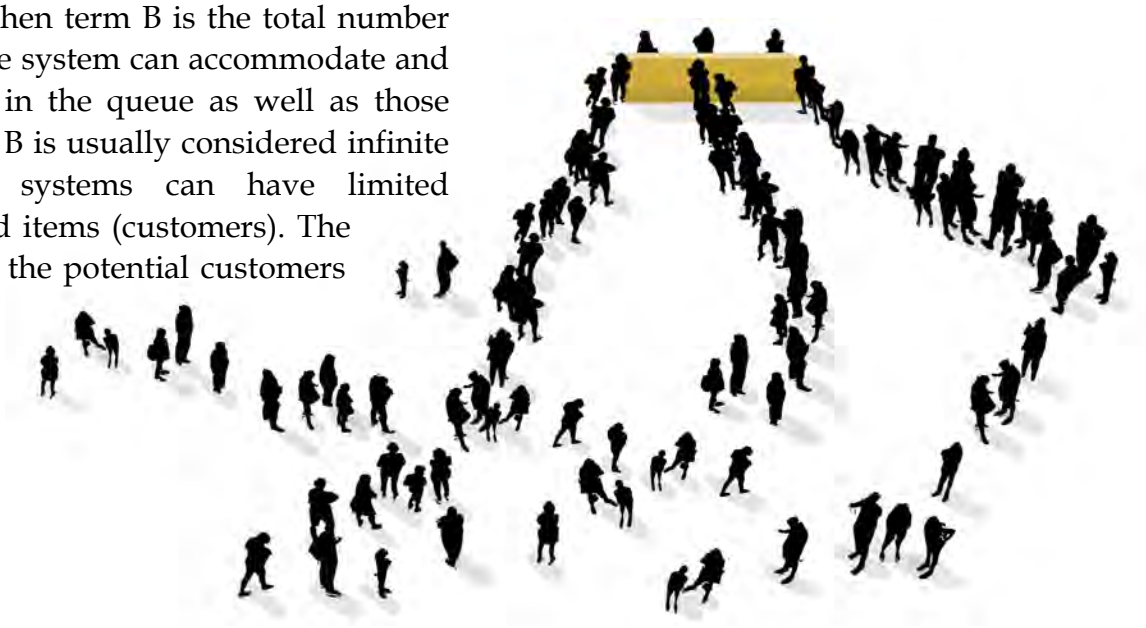
BY GAURIKA BINDAL

Have you ever been to a place where there are a number of queues? This is not uncommon in supermarkets, ticket booths, train stations, etc. So how does one choose which queue to join? This is where the queuing theory comes in. This theory looks upon a queuing system to arrive at answers to how long one will have to wait in a queue and such. A basic queuing system consists of the arrival process, the queue itself, the service process and the departure from the system. Queues themselves contain customers (or "items") such as people, objects, or information and form when there are limited resources for providing a service. All these affect the speed of the queue.

There is a shorthand notion used to denote all the parameters of a basic queue called the Kendall notion. It is written as A/S/c/B/N/D. The A describes the probability distribution that inter-arrival times follow that is the time interval between arrivals of items. The S term signifies the probability distribution of service time meaning, the time taken for an item to be serviced after leaving the queue. The c term specifies the number of servers in the system and it is assumed that all the servers are identical that is that all their work can be defined by S. Then term B is the total number of items that the system can accommodate and includes items in the queue as well as those being serviced. B is usually considered infinite though some systems can have limited capacity to hold items (customers). The term N defines the potential customers or the number of customers which could ever enter the queuing system. Lastly, D specifies the principle the queue follows, whether it is first-come-first-served or it is last-in-first-out.

One of the laws defining this theory is Little's Law. This law proven by the mathematician John Little states that the average number of items in a queue is proportional to the average rate at which the items arrive and the average amount of time they spend in it. Mathematically, the law is: $L = \lambda W$. L is the average number of items, λ is the average arrival rate of the items in the queuing system, and W is the average amount of time the items spend in the queuing system. The only drawback of this law is that it assumes the system is completely static and the variables characterizing it do not change over time.

This law is used to access a number of queuing systems and is very general despite using only three variables. It is useful in analyzing how the system performs over time or to understand how the queue is currently performing. This is used in a number of areas from the information technology, sales as well as manufacturing.

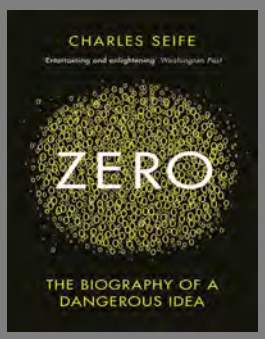


RECOMMENDATIONS

The Oxford Murders

The Oxford Murders is a British drama film directed by Alex de la Iglesias. This film is adapted from a novel by an Argentine mathematician Guillermo Martinez. The movie revolves around an American math student Martin who transfers to Oxford in the hope of persuading a celebrated philosopher Arthur Seldom to supervise his thesis, but the murder of Martin's landlady plunges the pair into a mathematical mystery. Could an `unsolvable' math problem provide the answers to the murderous conundrum?

Zero: The Biography of a Dangerous Idea



The Babylonians invented it, the Greeks banned it, the Hindus worshipped it, and the Church used it to fend off heretics. Over the centuries zero became the most important tool in mathematics. Zero follows this number from its birth as an Eastern philosophical concept to its struggle for acceptance in Europe and its apotheosis as the mystery of the black hole. Today, zero lies at the heart of one of the biggest scientific controversies of all time, the quest for the theory of everything. Elegant and witty, the book provides a compelling look at the greatest paradoxes of human thought.

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